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Intangible Capital, Corporate Earnings and the Business Cycle

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Abstract

Aggregate corporate profits are highly volatile and procyclical. Most dynamic general equilibrium models of the business cycle cannot deliver these basic features of the data. We develop a model of the U.S. economy in which firms expend resources to create intangible capital (IC), which is an additional input in their production technology. In keeping with the data, the model delivers profits that are many times more volatile than output. An estimated version of the model implies that IC investments are large and pro-cyclical. IC acts as a propagation mechanism, generating inertial responses to shocks. Overall, the model fits the aggregate data much better than a model without IC.

Keywords: Business Cycles; Profits; Bayesian Estimation; Intangible Capital.

JEL Codes: E3

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1 Introduction

Large swings in the profitability of U.S. corporations is an important aspect of the business cycle. For example, profits, as measured by aggregate real earnings of S&P 500 corporations, are about seven times as volatile as output and have a contemporaneous correlation of about 0.5. While investors fret over reported earnings and the financial media devote endless energy to anticipating corporate results during "earnings season", the macroeconomics literature on economic fluctuations is largely silent on the phenomenon.

A little reflection reveals that most popular models of business cycles are not consistent with the earnings data. The typical specification of the production technology calls for constant returns to scale in labour and physical capital services. With competitive markets this implies zero profits. One obvious way to generate profits is to assume decreasing returns to scale. This implies that profits move in proportion to output, yielding both a relative volatility number as well as a contemporaneous correlation coefficient equal to unity. Another obvious possibility is to assume that firms have some market power. Once again the typical formulation of imperfect competition implies a constant markup of price over marginal cost which implies the model cannot generate profits that are more volatile than output.¹

¹There do exist a few models with time varying markups which could potentially do the job, but a model with counter-cyclical markups (which is often the case) is unlikely to

In this paper we explore the idea that competitive firms may still generate profits because they produce intangible capital. Intangible capital is modeled as a third input in the technology for producing final goods in addition to labour and physical capital. If firm level investments in intangible capital are pro-cyclical, then the extra productivity unleashed by these investments can lead to profits rising more than output in periods of high activity. We embed this feature into an otherwise standard dynamic general equilibrium model and estimate the model using aggregate U.S. data. We find that the model generates aggregate profits that are much more volatile than output and are positively correlated with output as well. Simulations based on our estimates of model parameters reveal that earnings are roughly eight times as volatile as output and the correlation coefficient between the two series is 0.49. Investments in IC in the model are in fact procyclical and this cyclicity helps to explain the observed variation in real corporate earnings over the cycle.² When firms raise spending on IC, this comes at the expense of reduced work. An intriguing exception is the new study by Edmond and Veldkamp (2009) which posits a model with counter-cyclical markups but procyclical profit shares. However their model lacks capital accumulation so it is hard to assess the quantitative importance of the proposed mechanism in a full blown quantitative DGE model. The focus of the authors is also somewhat different. They discuss the relative volatility of the profit share as opposed to that of profits.

²The procyclicality of IC is reminiscent of the evidence that R&D expenditure is pro-

current profits but results in much higher profits in the quarters that follow.

Investments in intangible capital can be thought of as any expenditures by the firm (that are not included in physical capital investment) that raise it's ability to produce or that lower it's costs of production. Corrado et al (2006) mention spending on "innovative property (eg., R&D) and economic competencies as well as software and other computerized information..." as some types of intangible capital. This includes spending on strategic planning, spending on redesigning or reconfiguring existing products in existing markets, investments to retain or gain market share, and investments in brand names. They further divide R&D expenditures into a "scientific and non-scientific" category where the latter includes new product development in the services sector.

Recent evidence from the U.S. economy also suggests that these investments are a large and growing part of the economy. For example, Corrado et al (2006) report that the ratio of intangible to tangible investment increased from roughly 1.10 in the previous decade to over 1.3 since the turn of the century. This aggregate work is backed up by microeconomic studies such as Brynjolfsson et al (2002) which discuss the importance of firm specific cyclical. While R&D may be viewed as investment in intangible capital, most observers believe it to be a small part of the total expenditure on IC. Barlevy (2007) offers an alternate explanation for the procyclicality of R&D.

investments in (what they refer to as) organizational capital in determining the success of firms. A number of recent studies have also argued that investments in intangible capital by firms are important for understanding medium run observations on productivity and asset returns (McGrattan and Prescott 2007; Hall, 2001).

Despite the increased interest of economists in understanding the role played by, and magnitude of, intangible capital investments, little effort has been devoted to understanding the business cycle implications of an economy which devotes a significant amount of resources to the accumulation of intangible capital. We hope to fill this void. The goal is to go beyond a study of aggregate earnings dynamics and to understand how investments in intangible capital alter the dynamic response of the typical business cycle model to aggregate shocks as well as to understand how the business cycle influences the creation of intangible capital.

We estimate this model using Bayesian techniques and compare the performance of the model with intangible capital (IC) to one without IC. A number of interesting results emerge from this effort. We find that the ability of the *IC* model to explain aggregate output and hours data is many orders of magnitude higher than the model without IC. The estimation results indicate that in order to choose the *no-IC* model over the *IC* model, we need to assign a prior probability 7.11×10^{11} times larger to the *no-IC* model than

to the *IC* model. The model is an effective propagation mechanism, with output displaying considerably more inertia in response to shocks than the standard model (the AR(1) coefficient of output growth equals 0.3 vs. 0.003). This occurs because an increase in IC leads to future increases in productivity thus propagating shocks over time.

Our estimates imply that the steady state ratio of investments in intangible relative to tangible capital is about 0.75 which is less than the estimates reported by Corrado et al (2006) and slightly more than those reported by McGrattan and Prescott (2007). Firm investments in intangible capital allow it to earn a small profit in steady state of less than one percent of aggregate output which is in keeping with the earnings data.

As mentioned before, the model generates aggregate earnings behaviour that closely mimics the behaviour of the data.³ In contrast, the *no-IC* model predicts a correlation of unity between earnings and output as well as a relative standard deviation which is unity as well.⁴ A variant of the *no-IC* model

³In this study, we use S&P500 earnings as a proxy for real profits of firms. The earning data is taken from Shiller (2000). We also examined the corporate net cash flow (CNCF) series as an alternative proxy. The standard deviation of Hodrick-Prescott filtered (logged) quarterly CNCF relative to output is roughly 3.1 and the correlation with output is positive (0.55).

⁴Earnings in the *no-IC* model are calculated by allowing decreasing returns in the production technology for a fair comparison with the *IC* model. Strictly speaking our

where firms accumulate physical capital performs even worse, predicting a negative correlation with output. We think the ability of the *IC* model to explain earnings is important because it allows the model to distinguish itself from a host of other mechanisms to improve the fit of DGE models to observed output and hours series. Examples of the mechanisms we have in mind are based on modifications to the household side of the model such as changes in preferences (habit formation in leisure) or other ways to introduce dynamics in labour supply (learning effects, human capital etc.).⁵

1.1 Related literature

The idea that firms accumulate unobserved inputs that raise productivity has also been explored in the organizational capital literature. Here the accumulation of the unobserved input typically involves a learning-by-doing process by which firms accumulate knowledge as a by-product of production. Examples of dynamic general equilibrium models with this feature have been explored in closed and open economy settings in Cooper and Johri (2002), Johri and Lahiri (2008) and Johri (2009). A key difference between those estimates of the *no-IC* model imply constant returns to scale in production and therefore zero earnings.

⁵Examples of these include Perli and Sakellaris (1998), Chang et al (2002), Bouakez and Kano (2006), Jones, Manuelli and Siu (2005). While these may be important mechanisms in their own right, they cannot help explain the cyclical behaviour of earnings.

models and the IC model is that firms must expend valuable resources to produce intangible capital while the learning-by-doing process creates organizational capital as a by-product of production. We do borrow from that literature the idea that the contribution of past knowledge diminishes with time.⁶

Our IC model is related to, but different from, a number of recent ideas explored in the business cycle context using dynamic general equilibrium models. The first set of models study the implications of human capital for economic fluctuations. Early work on these ideas was published in Perli and Sakellaris (1998). These models typically model human capital as being accumulated by workers. Workers supply a joint input of human capital and raw labour to the production technology and get paid a wage commensurate with the return of the composite labour input. For example, Jones, Manuelli and Siu (2005) study the business cycle properties of an endogenous growth model with human capital accumulation. The planner can accumulate human capital by setting aside consumption goods as he would for investments in physical capital. Similarly Dejong and Ingram (2001) estimate a DGE model of skill acquisition using techniques similar to those used here. Unlike the pre-

⁶There is considerable evidence that this is indeed the case. See Benkard (2000) for examples as well as Cooper and Johri (2002) as well as Johri and Letendre (2007) for aggregate evidence.

vious study, skill or human capital is acquired purely as a function of time but not of goods. The modeling of human capital also differs in that it enters the production technology with a lag. In contrast with these models where agents expend resources on human capital, Chang et al (2002) invoke the notion of by-product learning-by-doing to accumulate human capital, again purely as a function of hours. Similarly Kim and Lee (2007) combine both aspects into their human capital accumulation equation. As mentioned earlier, our IC model focuses on decisions made at the firm level which lead to IC being a state variable in the firms problem allowing the firm to earn profits even though it operates in a competitive industry. Our model also differs from most of the above in that firms use both labour and physical capital to create new intangible capital.

Our model of intangible capital is also related to the endogenous R&D models of medium term fluctuations such as Comin and Gertler (2006). While the details of the model are very different, certain aspects of the R&D process have similarities to our intangible capital technology. In the Comin and Gertler model, agents invest consumption goods in a process that yields innovations, which if adopted, become new varieties of intermediate goods. As in Romer (1990), this leads to an increase in productivity in the final good technology.

Finally our *IC* model is similar to McGrattan and Prescott (2007). The

notion of unmeasured capital used in that paper is much broader than our notion of intangible capital but shares many similarities in how the technology for the creation of this capital is modeled. Nonetheless, the models differ in crucial respects. All expenses on IC in our model are accounted for and there is no "sweat equity" type notions used here. Moreover, we focus on explaining US business cycles generally while their focus is on the 1990's. Finally we provide estimates of our model while they focus on calibration.

The rest of the paper is organized as follows. Section 2 lays out the basic structure of our model economy. Section 3 discusses the econometric methodology and the data and then presents the empirical results. Section 4 concludes. Sensitivity analysis can be found in the appendix.

2 The Model Economy

In this section, we specify a decentralized dynamic general equilibrium model in which firms accumulate intangible capital. For convenience we refer to this model as the *IC* model which we will contrast with the standard model referred to as the *no-IC* model. While all agents in the economy operate in competitive markets, we will assume a single representative household and firm for convenience.

2.1 The Household's problem

The representative household maximizes its expected discounted utility over an infinite time horizon:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, \mathcal{B}_t). \quad (1)$$

Here β is the discount factor and the utility function in period t depends positively on contemporaneous consumption, C_t and labor supply, N_t . The variable \mathcal{B}_t represents a shock to preferences and follows a first-order autoregressive process with an iid error term:

$$\ln \mathcal{B}_t = \rho_b \ln \mathcal{B}_{t-1} + \epsilon_{bt} \quad (2)$$

In each period, the representative household supplies labor and physical capital to the firm, taking as given the wage rate w_t and the rental rate on capital, r_t^k . In addition, as the owner of the firm, the household receives any real profits earned by the firm, π_t . The sequence of budget constraints is given by

$$C_t + I_t = w_t N_t + r_t^k K_t + \pi_t. \quad (3)$$

The right-hand side of the budget constraint represents the sources of wealth: labor income $w_t N_t$; the return on the real capital stock, $r_t^k K_t$ and the profits earned by the firm. The left-hand side shows the uses of wealth: consumption

spending and investment (I_t) in physical capital. Investment augments the physical capital stock over time according to

$$K_{t+1} = I_t + (1 - \delta)K_t \quad (4)$$

where $\delta \in (0, 1)$ is a constant depreciation rate for physical capital.

Given initial values, the household chooses $\{C_t, N_t, I_t, K_{t+1}\}$, $t = 0, 1, 2, \dots$, to maximize the objective function (1) subject to the budget constraint (3) and the capital accumulation equation (4). The first-order conditions associated with this problem are:

$$w_t = \mathcal{B}_t \frac{U_{n,t}}{U_{c,t}} \quad (5)$$

$$1 = \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} (r_{t+1}^k + 1 - \delta) \right] \quad (6)$$

where $U_{c,t}$ and $U_{n,t}$ are, respectively, the marginal utility of consumption and marginal utility of leisure. Since the household problem is standard and well understood, we will omit any discussion of the optimality conditions and move on to the firm's problem.

2.2 The Firm's Problem

We assume that all production occurs at a single firm that behaves competitively and takes factor prices as given. The firm produces the final good according to the following constant returns to scale technology which uses

labour, physical capital and intangible capital as inputs:

$$y_t = (A_t u_t^n N_t)^\alpha (u_t^k K_t)^{1-\alpha-\varepsilon} Z_t^\varepsilon. \quad (7)$$

The presence of a third input, intangible capital, Z_t , in addition to the usual labor N_t and physical capital K_t is what distinguishes the model from the typical business cycle structure. Not all of the labor and capital hired by the firm goes to the production of the final good, y_t . The variables, u_t^n and u_t^k denote, respectively, the fraction of labor and physical capital which the firm allocates to output production. The remainder of labor and capital are used to produce new intangible capital. The technology shock, A_t , is assumed to follow a random walk with drift process:

$$\ln A_t = \gamma_a + \ln A_{t-1} + \epsilon_{at} \quad (8)$$

where ϵ_{At} are *iid* shocks.

The stock of intangible capital for the next period, requires labor, physical capital and intangible capital. The IC technology is given by

$$Z_{t+1} = \left[(A_t (1 - u_t^n) N_t)^{\alpha_1} ((1 - u_t^k) K_t)^{1-\alpha_1} \right]^{(1-\gamma)} Z_t^\gamma \quad (9)$$

where $\alpha_1(1 - \gamma)$ represents the elasticity of hours spent on creating IC in the current period with respect to IC in the next period. When $\alpha_1 = 0$, the firm allocates all of its labour to the production of the final good and when $\alpha_1 = 1$, physical capital is no longer used in intangible capital creation. The

other parameter $\gamma \in (0, 1)$, indicates that the contribution of past intangible capital decays the further back in time that it was created. This captures the idea that the relevance of knowledge falls with time as the economic environment undergoes change. This is consistent with the notion of organizational forgetting explored in Benkard (2000) and the depreciation of organizational capital discussed in the learning literature.⁷ We would expect the performance of the *IC* model to approach that of the *no-IC* model for values of γ close to unity. To see this, note that $\gamma = 1$ implies that intangible capital is constant over time. Note also that $\gamma = 0$ implies that intangible capital available to the firm at present makes no contribution to future levels of intangible capital. The productivity shock appears in equation (9) in order to ensure a balanced growth path and can be understood to imply that increases in the productivity of labour over time apply to both activities of the firm. This appears to be a reasonable assumption.⁸

⁷Note the IC technology may be viewed as a log-linear accumulation equation for IC with γ governing the depreciation rate of IC. Clarke (2006) suggests that the dynamics of organizational capital models with linear and log-linear accumulation equations is very similar. We expect the same to be true here.

⁸One might expect that there should be some randomness associated with the ability of the firm to create intangible capital. In practice, however, an additional shock in the IC equation gets confounded with the preference shock and cannot be separately identified when estimating the model. As a result we chose to leave it out.

In each period, the firm maximizes the present value of real profits:

$$\mathcal{X}_t = \max E_t \sum_{s=0}^{\infty} \Xi_t (y_{t+s} - w_{t+s}N_{t+s} - r_{t+s}^k K_{t+s})$$

subject to (7), and (9). where the variable $\Xi_t = \beta \frac{U_{c,t+1}}{U_{c,t}}$ is the appropriate endogenous discount factor for the firm. The first order conditions are given by:

$$w_t = \alpha \frac{y_t}{N_t} + \alpha_1(1 - \gamma)\lambda_t^Z \frac{Z_{t+1}}{N_t} \quad (10)$$

$$r_t^k = (1 - \alpha - \varepsilon) \frac{y_t}{K_t} + (1 - \gamma)(1 - \alpha_1)\lambda_t^Z \frac{Z_{t+1}}{K_t} \quad (11)$$

$$\alpha \frac{y_t}{u_t^n} = \alpha_1(1 - \gamma)\lambda_t^Z \frac{Z_{t+1}}{1 - u_t^n} \quad (12)$$

$$(1 - \alpha - \varepsilon) \frac{y_t}{u_t^k} = (1 - \gamma)(1 - \alpha_1)\lambda_t^Z \frac{Z_{t+1}}{1 - u_t^k} \quad (13)$$

$$\lambda_t^Z = E_t \left[\frac{\Xi_{t+1}}{\Xi_t} \left(\varepsilon \frac{y_{t+1}}{Z_{t+1}} + \gamma \lambda_{t+1}^Z \frac{Z_{t+2}}{Z_{t+1}} \right) \right] \quad (14)$$

where λ_t^Z is the Lagrange multiplier associated with equation (9). Equations (10) and (11) differ from the typical conditions in that the firm will not equate the marginal product of labor and physical capital, respectively, to their factor prices. Rather, the prices will be higher than the marginal products. This occurs because only a part of labour and capital is used in production, the rest is used to produce intangible capital which in turn raises production and hence profits in the future. This dynamic consideration facing the firm when it decides how much labour and capital to hire shows up in the additional term involving Z_{t+1} that appears on the right hand side of both conditions.

Equations (12) and (13) state that the firm should allocate inputs from the production of final good to intangible capital in such a way that the marginal decrease in output is exactly equal to the value of the marginal increase in intangible capital made available to the firm as a result of the switch. Replacing (12) and (13) in (10) and (11) respectively yields (15) and (16) below. Since u^n and u^k are positive fractions, it is clear that factor prices exceed their marginal products in final output production. Note also that the u^i 's act as time varying wedges between factor prices and marginal products.⁹

$$w_t = \frac{1}{u_t^n} \cdot \alpha \frac{y_t}{N_t} \quad (15)$$

$$r_t^k = \frac{1}{u_t^k} \cdot (1 - \alpha - \varepsilon) \frac{y_t}{K_t} \quad (16)$$

Equation (14) establishes the marginal value of an extra unit of intangible capital to the firm. The benefit comes not only from the extra production of final good made possible but also from the additional intangible capital that can be produced in the future.

Recall that firm earnings or profit each period is given by

$$E_t = y_t - w_t N_t - r_t^k K_t. \quad (17)$$

⁹The wedge between marginal product of labour and wages in the data is often interpreted as evidence of monopoly power. In steady state, our model estimates would imply a markup of $\frac{1}{0.79} = 1.27$ even though the firm behaves competitively.

Substituting (15) and (16) for factor prices yields

$$E_t = y_t \left(1 - \frac{1}{u_t^n} \cdot \alpha - \frac{1}{u_t^k} \cdot (1 - \alpha - \varepsilon) \right). \quad (18)$$

It is clear from (18) that the time varying nature of the factor proportions (u^i 's) are crucial for breaking the tight link between earnings and output implied by the model without intangible capital.

Defining the firm's investment in intangible capital as I_t^z where

$$I_t^z = w_t N_t (1 - u_t^n) + r_t^k K_t (1 - u_t^k), \quad (19)$$

and rearranging this using (15) and (16) we can write the relationship between output and investment in IC as

$$\frac{I_t^z}{y_t} = \left(\frac{1}{u_t^n} - 1 \right) \cdot \alpha + \left(\frac{1}{u_t^k} - 1 \right) \cdot (1 - \alpha - \varepsilon). \quad (20)$$

Clearly investments in IC are increasing in output and decreasing in the share of factors allocated to output, u 's. If these u 's are procyclical then they work towards generating counter-cyclical investment in IC, though we would expect the positive relationship between output and investment in IC to dominate. The above equation (19) also allows us to tease out the relationship between the firm's earnings and investments in its IC:

$$E_t = \varepsilon y_t - I_t^z. \quad (21)$$

Equation (21) brings out the trade-off facing the firm nicely. Investments in intangible capital allow the firm to become more efficient in the future,

raising future profits, however, this comes at the cost of reducing current period profits by diverting resources away from the production of the final good.

The *no-IC* model is just a special case of the *IC* model so we do not discuss it in detail. The efficiency conditions would be the same as here as long as u_t^i 's were equal to unity for all t . In addition (9) would be eliminated.

2.3 Equilibrium

A competitive equilibrium consists of sequences of allocations $\{C_t, n_t, I_t, K_{t+1}, E_t, u_t^n, u_t^k, Z_{t+1}, y_t\}_{t=0}^\infty$, and prices $\{w_t, r_t^k\}_{t=0}^\infty$ such that, taking as given K_0, Z_0 and exogenous processes $\{A_t, \mathcal{B}_t\}_{t=0}^\infty$:

- $\{C_t, n_t, I_t, E_t, K_{t+1}\}_{t=0}^\infty$ solve the household problem
- $\{u_t^n, u_t^k, Z_{t+1}, y_t\}_{t=0}^\infty$ solve the firm's problem
- Market clearing conditions for goods, labor and physical capital are satisfied.

2.4 Intangible Capital in Steady State

In this section we discuss some properties of intangible capital in steady state and the impact of varying key parameters associated with the intangible capital process.

The ratio of investment in intangible capital to output $\frac{\bar{I}^z}{\bar{y}}$ and the ratio of the investment in intangible relative to tangible capital $\frac{\bar{I}^z}{\bar{I}}$ in steady state are now, respectively, given by

$$\frac{\bar{I}^z}{\bar{y}} = \beta\varepsilon \frac{1-\gamma}{1-\beta\gamma} \quad (22)$$

$$\frac{\bar{I}^z}{\bar{I}} = \frac{\bar{r}\beta\varepsilon \frac{1-\gamma}{1-\beta\gamma}}{\bar{z} \left(1 - \alpha - \varepsilon + (1 - \alpha_1)\beta\varepsilon \frac{1-\gamma}{1-\beta\gamma} \right)} \quad (23)$$

where \bar{z} denotes the constant growth rate of output and $\bar{r} = \frac{\bar{z}}{\beta} + \delta - 1$ denotes the steady state interest rate.

According to equation (22), the ratio of the investment in IC to output, I^z/Y depends only on the parameters ε and γ but not on α_1 . Figure 1 shows how this ratio varies with respect to these two parameters. The results indicates that ε is the key parameter in determining the value of I^z/Y while γ only has a marginal effect. I^z/Y increases strongly with ε because this parameter controls the contribution of intangible capital to output.

A number of studies for the U.S. have reported quite a large range for the ratio of investment in intangible relative to tangible capital (see, for example, Corrado et al, 2006 and McGrattan and Prescott, 2007). We find that our model can imply a wide range of values for this ratio as key parameters of the IC process are varied. Figure 2 displays these relationships. The panels plot, respectively, the values of the steady state ratio as ε , α_1 and γ vary. We find that the ratio of the two investments is increasing strongly in both ε and

α_1 and is decreasing in γ . To see the impact of ε on the results, note first that physical capital contributes to output both directly via the production function and indirectly as an input in the creation of intangible capital. Next note that as the share of intangible capital in output increases, the share of physical capital in output becomes smaller due to the restriction of constant return to scale. Thus, as ε increases, the direct effect of intangible capital in goods production becomes more important while the direct effect of tangible capital in output is weakened. This fact makes investment in intangible capital more favourable over investment in physical capital, which leads to a larger \bar{I}^z/\bar{I} . By contrast, as γ increases, the current stock of intangible capital contributes more to future intangible capital. Therefore, the firm has less incentive to invest in intangible capital, which leads to a fall in \bar{I}^z/\bar{I} . Finally, as α_1 gets larger, physical capital contributes less to the production of Z_{t+1} relative to Z_t , which dampens the indirect effect of physical capital on output through future intangible capital. So it makes sense to invest more in intangible relative to physical capital. Figure 2 suggests that for plausible values of parameters, our model predicts that this ratio will be less than unity.

3 Empirical Method and Results

In this section we discuss our methodology for estimating and evaluating the empirical performance of two competing models. We make use of Bayesian methods which have become popular in the DSGE literature.¹⁰ Note that the equilibrium system of a DSGE model can be linearly approximated around its stationary steady-state in the form of

$$AE_t(\hat{\mathbf{x}}_t|\mathbf{I}_t) = \mathbf{B}\hat{\mathbf{x}}_t + \mathbf{C}(F)E(\xi_t|\mathbf{I}_t) \quad (24)$$

where $\hat{\mathbf{x}}_t$ is a vector of endogenous variables¹¹, $E_t(\hat{\mathbf{x}}_t|\mathbf{I}_t)$ is the expectation of $\hat{\mathbf{x}}_{t+1}$ given period t information, ξ_t is a vector of exogenous stochastic processes underlying the system, and $\mathbf{C}(F)$ is a matrix polynomial of the forward operator F . The solution of the log-linearized system (24) can be written in the following state-space form:

$$\hat{s}_{t+1} = \mathbf{P}\hat{s}_t + \mathbf{C}_1\xi_{t+1} \quad (25)$$

$$\hat{y}_t = \mathbf{Q}\hat{s}_t \quad (26)$$

where the vector $\xi = \begin{bmatrix} \hat{\xi}_{At} \\ \hat{\xi}_{pt} \end{bmatrix}$ contains technology and preference innovations.

Then we update the state-form solution by adding a set of measurement equa-

¹⁰See, for example, Rabanal and Rubio-Ramírez, (2005); Lubik and Schorfheide, (2007); Del Negro and Schorfheide, (2008)

¹¹For any stationary variable x_t , we define $\hat{x}_t = \left(\frac{x_t - \bar{x}}{\bar{x}}\right)$ as the percentage deviation from its steady-state value, \bar{x} .

tions which link the observed time series to the vector of unobserved state variables. We further use the Kalman filter to evaluate the likelihood function of the state-space form solution and combine the likelihood function with our specified prior knowledge about these deep parameters to form the posterior distribution function. The sequence of posterior draws can be obtained using Markov Chain Monte Carlo (MCMC) methods. We use the random-walk Metropolis-Hasting algorithm as described in Schorfheide (2000) to numerically generate the Markov chains for the structural parameters.

3.1 Data

We use quarterly U.S. data taken from the Federal Reserve Bank of St. Louis' FED database. The data sample consists of seasonally adjusted quarterly time series, from 1959:1 to 2008:3, on total hours for non-agricultural industries and growth rate of real GDP in chained 2000 dollars. Both series are expressed in per capita terms by dividing by the civilian non-institutional population, ages 16 and over.

3.2 Specification of Priors

To specify our priors, we use information about key ratios in steady state to pin down prior means. This is particularly important for ε , α_1 and γ as

there is little guidance in the literature about reasonable values to use. These ratios are the capital output ratio and the labour share in steady state which are given by the following equations

$$\frac{\bar{K}}{\bar{y}} = \frac{\bar{z}}{\bar{r}} \left(1 - \alpha - \varepsilon + (1 - \alpha_1)\beta\varepsilon \frac{1 - \gamma}{1 - \beta\gamma} \right) \quad (27)$$

$$L\bar{S}H = \alpha + \alpha_1\beta\varepsilon \frac{1 - \gamma}{1 - \beta\gamma} \quad (28)$$

Figure (4) displays the sensitivity of the capital-output ratio and the labour share with respect to these intangible capital parameters. As we can see, γ does not have a big influence on these ratios. Therefore, we assign a common uniform prior distribution to γ , with a lower bound of 0 and an upper bound of 1. In contrast, to assign prior means to ε and α_1 , we choose parameter values such that the capital-output ratio and the labour share are calibrated to 10.1 and 0.6, respectively. Note that the steady state ratios are fairly sensitive to the values of these two parameters. Based on this fact, we choose relatively tight priors. Specifically, we choose a beta distribution on ε with a mean of 0.09 and standard error of 0.05. The 95 percent confidence interval for ε extends from 0 to 0.71. We also give a beta distribution to α_1 with a mean of 0.55 and standard error of 0.2. The 95 percent confidence interval for α_1 extends from 0 to 0.99.

We assign a Beta distribution with mean of 0.8 and standard deviation of 0.1 to ρ_p the autoregressive coefficients of the stationary exogenous process.

We choose uninformative inverse gamma distributions for the precision of the structural shocks, $\{\sigma_A, \sigma_p\}$.

Table 1 presents the marginal prior distributions for the structural parameters. The choice of prior distributions for parameters reflect restrictions on their natural domain, such as non-negativity or interval restrictions. In addition, the priors on the structural parameters are assumed to be independent of each other, which allows for easier construction of the joint prior density used in the MCMC algorithm. Thus, the joint distribution is assumed to be the product of independent prior distributions with

$$p(\Theta|\mathcal{M}_i) = p(\alpha|\mathcal{M}_i)p(\alpha_1|\mathcal{M}_i)p(\eta|\mathcal{M}_i) \dots p(\omega_n|\mathcal{M}_i) \quad (29)$$

The depreciation rate of capital δ is assumed to follow a Beta distribution with a mean of 0.025 and standard error of 0.003. The prior for α is described by a Beta distribution with a mean of 0.55 (which implies a labor share of 0.6) and standard error of 0.05. Regarding the labor supply elasticity, we assume ϕ follows a Gamma distribution with a mean of 2 with a standard error of 0.5. As these deep parameters are largely in line with the literature, we use tight priors¹² to make the estimated model a-priori comparable to those in the literature. In all models, we calibrate the discount factor β equal

¹²The prior variance were chosen to reflect a reasonable degree of uncertainty over the calibrated values of parameters.

to 0.99, which implies a steady-state annually real interest rate of 4 per cent.

3.3 Posterior Estimates

Table 2 reports the posterior distribution of parameters based on 250,000 draws from two independent Markov chains. Of special interest here are the intangible capital parameters. The estimate of ε , in the *IC* model is approximately 0.173 while that of γ is 0.6. Our estimate of α_1 equal to 0.85 suggests that labour is much more important in creating intangible capital than physical capital. The other structural parameters defining preferences and technology are estimated to be roughly of the same magnitude in both models, and the posterior means are consistent with a number of other calibrated and estimated DSGE models. The estimated posterior mean of α is 0.54, which implies the labor share in output equals 0.67. Our estimate of the quarterly depreciation rate of tangible capital δ and the determinist growth rate γ_a are 2.2% and 0.34%, respectively. The estimated process for the stationary preference shock, \mathcal{B}_t , is highly persistent with the standard deviation of innovations equal to 0.5%. The posterior standard deviation of a permanent technology shock is 1.2. These estimates imply a capital output ratio of 9.11. Our estimates suggest that the steady state share of profits (E/Y) is 0.42%. This is close to the mean value in US data of 0.76%. In addition,

the implied ratio of intangible to tangible capital investment is 0.748. As a comparison, McGrattan and Prescott (2007) report this ratio equals 0.42.

The posterior means of the structural parameters in the *no-IC* model are consistent with estimates reported in previous studies (see, for example, Chang et al, 2002; Ireland, 2004).

3.4 Model Fit and Marginal Data Densities

In this section, we compare the *no-IC* model to the *IC* model in terms of how well they fit the aggregate data. Given the estimates of the two competing models, we conduct a comparison of the overall time series fit between the DSGE models and a comparable a-theoretical VAR model. Table 3 reports the marginal data densities¹³ and posterior odds ratios. The results indicate that in order to choose the *no-IC* model over the *IC* model, we need to assign a prior probability 7.11×10^{11} times larger to the *no-IC* model than to the *IC* model. This result indicates that introducing intangible capital into our standard DSGE models leads to a significant improvement in the ability of these models to fit the aggregate data. The time-series fit of *IC* model remains inferior to that of the VAR(4) as is typically found in the literature.

¹³The marginal data densities are computed using Geweke's modified harmonic mean estimator and a Markov chain of 150,000 draws for each specification of DSGE models.

3.5 Explaining key features of business cycles

3.5.1 Propagation

As emphasized in numerous studies (e.g. Cogley and Nason, 1995 and King et al, 1988), successful business cycle models must contain effective mechanisms to propagate shocks over time. It is well known that the *no-IC* model falls short on this account. For example, output growth is positively autocorrelated over short horizons and weakly autocorrelated over longer horizons (Cogley and Nason, 1995 and Chang et al, 2002). The *IC* model is able to propagate shocks since firms respond to an increase in productivity by acquiring more intangible capital thus raising future productivity.

In Table 4, we compare the autocorrelations of output growth predicted by the estimated *no-IC* and *IC* models to those of US data for the period 1960:1 to 2008:3. The results clearly show that the *no-IC* model predicts autocorrelations of output growth to be essentially zero, while the *IC* model is capable of generating positive autocorrelations of output growth, which match the autocorrelations of the data quite well over short horizons. In order to formally evaluate the models using model-based and observed autocorrelations, we specify a posterior expected loss function, L_q .¹⁴ The measure

¹⁴See Schorfheide (2000) for a detailed discussion of these loss functions and their interpretations

of loss reported in Table 4 confirms that the *IC* model ($L_q = 0.019$) does much better than the *no-IC* model ($L_q = 0.149$) in explaining output growth autocorrelations.

3.5.2 Corporate Earning Dynamics and Intangible Capital

As discussed in the Introduction, a key feature of the *IC* model is its ability to explain the behaviour of real corporate earnings over the business cycle. Earnings in S&P 500 corporations are much more volatile than output and positively correlated with it. As shown in Table 5, the *IC* model is able to deliver both features of the earning's data.¹⁵ In the *no-IC* model, firms generate no profits due to the assumption of constant returns to scale in labour and capital. One way to allow a comparison is to impose decreasing returns to scale on the production technology. We do this by keeping the return on labour and capital equal to that used in the *IC* model.¹⁶ Not surprisingly, earnings generated by the *no-IC* model using this method are perfectly correlated with output and have the same volatility as output.

¹⁵Table 5 reports the Hodrick-Prescott filtered moments. Note that the moments of corporate earning are sensitive to the filtering method. The corresponding linearly detrended correlation coefficient between earning and output is 0.50 in the model and 0.52 in the data. The relative standard deviation numbers are, respectively, 5.51 in the model and 6.78 in the data. The dynamic correlations shows similar lead-lag pattern.

¹⁶In decreasing returns to scale case, we choose $\alpha = 0.59$ and $\theta = 0.84 - \alpha$.

In order to give the model without intangible capital a better chance to account for the behaviour of earnings we consider a modified version of the *no-IC* model where the firm, rather than households, accumulate physical capital. As a result of this change, the firm's earnings are now given by

$$E_t = y_t - w_t n_t - I_t \quad (30)$$

where the notation is the same as before. Since the firm will not pay all its output to the factor's of production, there will be some earnings generated. In Figure 5 we plot the cross-correlation of earnings in period t with both leads and lags of output for the data, the *IC* model as well as the version of *no-IC* model described above. While the *IC* model does a fair job of capturing the lead lag relationship, the alternative model predicts a strongly negative relationship which is completely counter-factual. In Figure 6, we plot simulated earnings from the *IC* model against the data. The figure shows that the model predictions are quite good. The overall correlation between the two series is 0.53.

To further explore the role of intangible capital in explaining earning dynamics, we display the relationship between the relative standard deviation and the contemporaneous correlation of earnings with respect to output and key intangible capital parameters in Figure 7 and 8, respectively. Figure 7 indicates that the volatility of earnings is increasing in ε and α_1 but decreasing

in γ . The relationship between ε and earnings is clear from

$$E_t = \varepsilon y_t - I_t^z. \quad (31)$$

Note also that given the volatility of output, the volatility of earnings depends on how strongly investment in intangible capital responds to shocks. Figure 9 shows how the standard deviation of I_t^z is influenced by α_1 and γ . Recall that as γ goes to one, the model approaches one in which intangible capital becomes constant. This occurs because of the constant returns to scale assumption imposed on the intangible capital equation. As γ rises, the contribution of labour and capital to intangible capital accumulation falls towards zero. With no change in intangible capital, earnings can only respond to movements in output, leading to a fall in the volatility of earnings. Turning to α_1 , we note that as α_1 rises, the contribution of labour to the creation of new intangible capital increases while the contribution of physical capital falls. Since the productivity shock is labour augmenting, an increase in α_1 makes the shock more potent in creating intangible capital. This induces the firm to transfer a bigger proportion of an already rising total hours over to the creation of intangible capital. Since hours respond more strongly to shocks than physical capital, the rising contribution of hours on I_t^z overwhelms the falling contribution of capital leading to a larger response of I_t^z . As before, bigger movements in investment in intangible capital translate into more

volatility of earnings.¹⁷ Figure 8 shows the contemporaneous correlation between earnings and output as the same parameters are varied. This moment, however, is decreasing in ε and α_1 but increasing in γ . Comparing Figure 7 and 8, a trade-off between volatility and correlation is clearly visible as any one parameter changes. This is to be expected given that any attempt to increase investment in intangible capital will result in lower contemporaneous earnings (but higher future profits).

3.5.3 Standard Business Cycle Moments

In this section, we discuss the performance of the model with respect to the typical second moments reported in the business cycle literature. Table 5, reports these moments for the *IC* and *no-IC* models and contrasts them with their unconditional data counterparts. All the business cycle statistics reported in Table 5 are calculated using stationary cyclical deviations based on the Hodrick-Prescott filter and are calculated using the estimated shocks for each model.

Both models underpredict the volatility in consumption and investment

¹⁷Detailed sensitivity analysis with impulse responses (not shown here) confirm that I_t^z responds more strongly as α_1 is increased. This increase in volatility occurs mostly in response to productivity shocks and to a lesser extent with preference shocks. It also reveals that u^n falls more if α_1 is raised.

relative to output but the *IC* model does better on both accounts. Both models get the relative volatility of hours and wages roughly right. The contemporaneous correlations of the above macroeconomic variables with output are also similar to each other and to the data.

Figure 10 reports the lead lag pattern in the cross-correlation of hours, consumption and investment with output. The *IC* model clearly follows the patterns seen in the data.¹⁸

3.6 Impulse-Response Dynamics

In this section we display and discuss the impulse response of key variables of the model to the two shocks. Our goals are two-fold. First, we wish to compare the model responses to the data. Second, we wish to further explore the role played by the presence of intangible capital on the dynamics of the model.

3.6.1 A brief comparison with a VAR

To shed more light on how well the *IC* model captures the dynamics of output and hours worked, we compare the impulse-responses from both structural

¹⁸The correlation patterns of the *no-IC* model are similar and not reported for this reason.

models with the counterparts from the estimated a-theoretical VAR.¹⁹ The first column of Figure 11 presents the posterior means of the impulse-response of output and hours worked to a one-standard deviation increase in labour augmenting productivity generated by the *no-IC*, *IC* and VAR models, respectively. The two models generate completely different initial dynamics in response to the technology shock. In the *IC* model, both output and hours worked display an inertial response which tracks the shape of the VAR-based counterpart more closely than the *no-IC* model. In particular, the response of output and hours rises for the first few quarters before peaking which is similar to the VAR based response. This feature is missing in the absence of intangible capital.

The second column of Figure 11 reports the posterior means of the impulse response to a one-standard deviation shock to the transitory process in each model. As documented by the literature, (see, for example, Blanchard and Quah, 1989; Cochrane, 1994; Cogley and Nason, 1995) the VAR response of output to the transitory shock exhibits a pronounced hump-shape and trend reverting path. The *no-IC* model lacks this hump in output, while the *IC* model produces a pronounced hump-shaped output response, which matches

¹⁹To enable a comparison between the DSGE models and the a-theoretical VAR model, we employ Blanchard and Quah's (1989) method to identify the permanent and transitory shocks in the VAR.

the VAR response fairly well in the first few quarters. The response of hours worked to a transitory shock in the *IC* model also generates a small hump-shaped response while the *no-IC* model displays monotonic convergence of hours worked towards its steady state.

3.6.2 The Role of Intangible Capital in Propagating Shocks

In order to understand how the presence of intangible capital changes the response of the basic DSGE model to shocks, we plot the response of investment in intangible capital in Figure 12 along with output and earnings. The top panel plots the response to a permanent technology shock while the bottom panel plots the response to a transitory shock.

The advent of a positive technology shock leads to a permanent rise in the productivity of labour. This induces the firm to hire more hours, as we saw in the previous section. Since productivity has increased in both the creation of goods as well as intangible capital, there is an incentive to expand both output as well as invest more in intangible capital. Given our estimates of $\alpha_1 > \alpha$, the shock has a bigger impact on the IC equation than the production function so the firm also chooses to substitute some labour towards the creation of new intangible capital. This is achieved by reducing u_t^n below steady state levels(not shown). As a result, the firm ramps up investment in intangible capital slightly more than output. This investment in

future productivity occurs even though earnings temporarily fall below steady state. Eventually, the ensuing rise in intangible capital becomes sufficiently large that the extra productivity unleashed is sufficient to pay for the extra investment in intangible capital and yet allow earnings to rise above steady state.

A very similar pattern is evident in the bottom panel where the firm is induced to hire more labour because of a fall in wages (not shown) driven by the preference shock. The large increase in hours worked at the firm allows an expansion of both output and investment in intangible capital. One difference relative to the top panel is that both variables rise by about the same amount. The firm raises u_t^n thus diverting resources towards production. Once again earnings fall below steady state for a couple of periods but this is offset by many future quarters of above steady state earnings. Figure 12 also illustrates why the *IC* model can lower the correlation of earnings and output below unity.

The impact of investment in intangible capital on endogenous productivity dynamics of the economy becomes even more clear by studying the impulse response of the Solow Residual in the *IC* model. Defining the Solow Residual in the usual way it can be easily shown that it is composed of both

endogenous and exogenous variables:

$$SR_t = \frac{y_t}{N_t^\alpha K_t^{1-\alpha-\varepsilon}} = (A_t u_t^n)^\alpha (u_t^k)^{1-\alpha-\varepsilon} Z_t^\varepsilon. \quad (32)$$

The Solow residual now varies with movement in either intangible capital or the share of factors used in goods production even in the absence of any productivity shock. Figure 13 shows the response of the Solow residual to a preference shock in the lower panel and to a productivity shock in the upper panel. The lower panel shows the Solow residual dips on impact as the firm diverts resources towards accumulating intangible capital and away from production of the final good (a fall in u_t^n). This leads to a persistent rise in productivity as the extra intangible capital goes to work producing more final goods. It is clear that the model generates a highly persistent endogenous productivity response. This extra productivity is the payoff for giving up some earnings in the initial periods. According to equation (32), three components contribute to the impulse dynamics of Solow Residual. It is worth noting that the shares of factors used in good production do not move much and thus movement in intangible capital is the primary ingredient in the dynamic behaviour of Solow Residual after the initial period. The response of the Solow Residual to productivity shocks is similar in that SR_t rises above A_t and only slowly returns to its new steady state level.

4 Conclusion

An important feature of business cycles is that profits increase in booms and fall in recessions. Using real earnings data for S&P 500 corporations as a proxy for aggregate profits we find that earnings are roughly seven times more volatile than aggregate output. Most business cycle models cannot deliver this feature of the data. In this paper we explore the idea that firms generate profits because they produce intangible capital. Intangible capital is modeled as a third input in the technology for producing final goods in addition to labour and physical capital. Firms can invest in creating intangible capital by diverting resources away from the production of final goods. We embed this model of intangible capital into an otherwise standard DSGE model and ask if it can deliver reasonable predictions about aggregate profits without sacrifices on the usual metrics used to evaluate business cycle models.

The model is estimated using bayesian techniques and provides a significantly better fit with aggregate U.S. time series data on hours and output than the model without intangible capital. Simulation results from an estimated version of our model are similar in many respects to a number of recent models which also show an improvement in fit as well as an improved ability to propagate shocks and generate inertial responses which mimic a theoretical VAR-based impulse response functions. What distinguishes our

model from many of these exercises is the ability of our model to explain the dynamics of aggregate real corporate earnings. The intangible capital model can generate earnings volatility which is roughly the same magnitude as the data. It can also deliver less than perfect co-movement between earnings and output, which is another feature of the data as well as broadly capture the lead-lag patterns of these variables. In the absence of intangible capital, the model fails to replicate these features of the data.

To our knowledge, this paper is the first to provide aggregate estimates for a model with intangible capital, a phenomenon which is inherently hard to measure directly. We find that investments in intangible capital are procyclical and substantial in magnitude and play a major role in generating endogenous movements in productivity over the cycle. Our estimates imply that investments in intangible capital are roughly three-fourths the size of investments in physical capital.

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TABLE 1: Prior Distribution for the Structural Parameters

Parameter	Range	Density	Mean	S.D.
IC Parameters, Prior:				
ε	$[0, 1]$	Beta	0.09	0.05
γ	$[0, 1]$	Uniform	-	-
α_1	$[0, 1]$	Beta	0.55	0.2
Additional Parameters:				
α	$[0, 1]$	Beta	0.55	0.05
γ_a	\Re	Normal	0.005	0.005
δ	$[0, 1]$	Beta	0.025	0.003
ϕ	\Re^+	Gamma	2	0.5
ρ_p	$[0, 1]$	Beta	0.8	0.1
σ_A	\Re^+	Inverse Gamma	0.02	∞
σ_p	\Re^+	Inverse Gamma	0.02	∞

TABLE 2: Posterior Estimates for the Structural Parameters

Parameter	<i>no-IC</i> model		<i>IC</i> model	
	Post. Mean	S.D.	Post. Mean	S.D.
ε	-	-	0.173	0.034
γ	-	-	0.592	0.075
α_1	-	-	0.848	0.086
α	0.614	0.019	0.535	0.020
γ_a	0.0037	0.001	0.0034	0.001
δ	0.024	0.003	0.022	0.003
ϕ	0.863	0.018	0.887	0.018
ρ_p	0.976	0.010	0.978	0.010
σ_A	0.010	0.001	0.012	0.001
σ_p	0.005	0.000	0.005	0.000

Notes: The posterior means are calculated from the output of the Metropolis-Hastings algorithm and S.D. denotes the standard deviation.

TABLE 3: Goodness of Fit

Statistic	<i>no-IC</i> model	<i>IC</i> model	VAR(4)
Prior probability, $\pi_{i,0}$	1/3	1/3	1/3
Log marginal data density	1215.07	1242.36	1265.94
Posterior odds ratio	1.00	7.11×10^{11}	1.23×10^{22}
Posterior probability, $\pi_{i,T}$	8.08×10^{-23}	3.89×10^{-11}	1.00

Notes: Marginal data densities for the DSGE models are computed by Geweke's (1999) modified harmonic-mean estimator. The marginal data density of the VAR is computed via Monte Carlo approximation of one-step-ahead predictive densities.

TABLE 4: Autocorrelation Statistics

Statistic	Lag	<i>no-IC</i> model	<i>IC</i> model	VAR(4)
(A) Output Growth, $\text{corr}(\Delta \ln Y_t, \Delta \ln Y_{t-j})$:				
	1	0.003	0.301	[0.217, 0.322]
	2	0.003	0.191	[0.189, 0.207]
	3	0.003	0.119	[−0.026, 0.067]
	4	0.002	0.073	[−0.036, −0.026]
L_q risk	1 – 4	0.149	0.013	

TABLE 5: Second-Order Unconditional Moments

Moments	US Data	<i>no-IC</i> model	<i>IC</i> model
σ_y	1.62	1.61	1.61
σ_c/σ_y	0.78	0.43	0.51
σ_i/σ_y	4.49	2.52	2.93
σ_n/σ_y	1.13	1.16	1.16
σ_w/σ_y	0.55	0.55	0.58
σ_{earn}/σ_y	7.36	1.00	8.50
$\text{corr}(c, y)$	0.87	0.97	0.94
$\text{corr}(n, y)$	0.88	0.88	0.88
$\text{corr}(i, y)$	0.89	0.99	0.98
$\text{corr}(earn, y)$	0.49	1.00(−0.97)	0.15

Notes: The statistics are based on Hodrick-Prescott filtered quarterly U.S. data for the period 1965:1-2008:3. All variables are in logarithms.

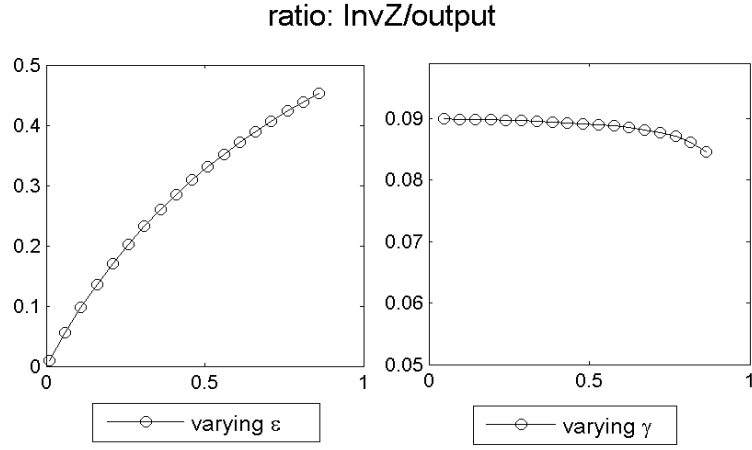


FIG. 1: Ratio: I^z/Y

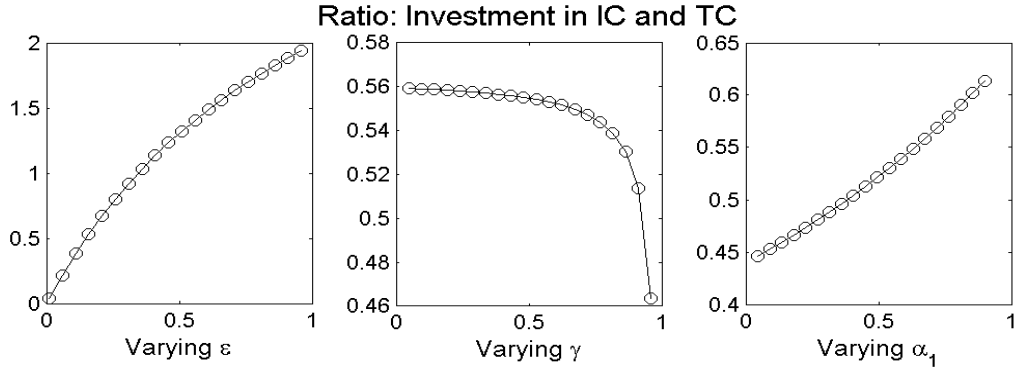


FIG. 2: Investment Ratios at the steady state

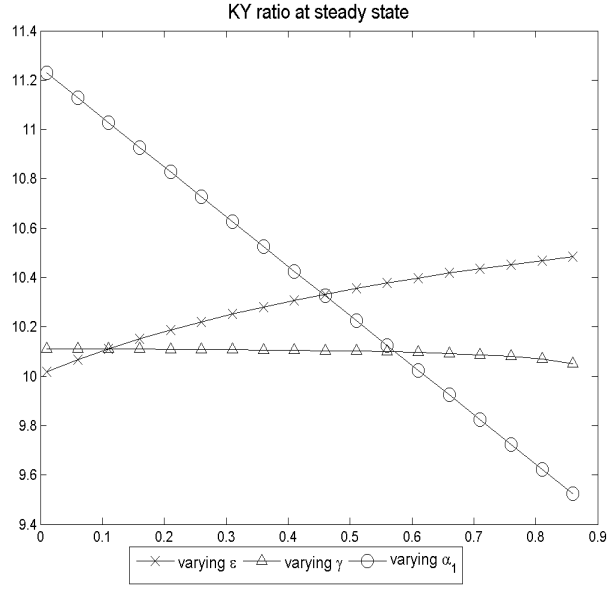


FIG. 3: Capital-output ratio at steady state.

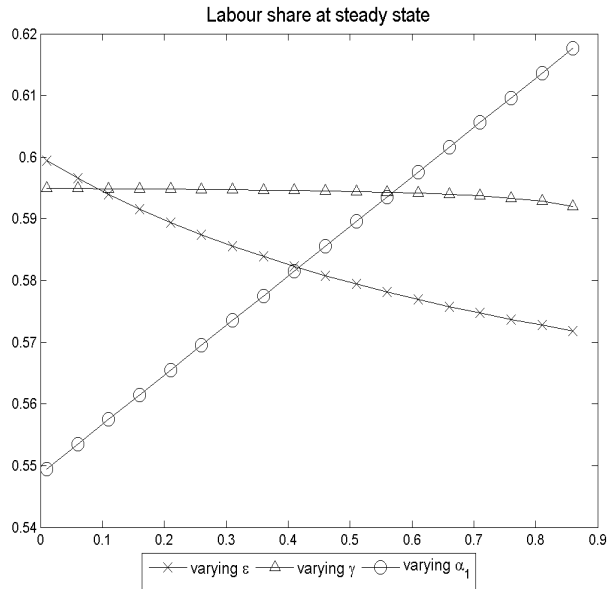


FIG. 4: Labour share at steady state.

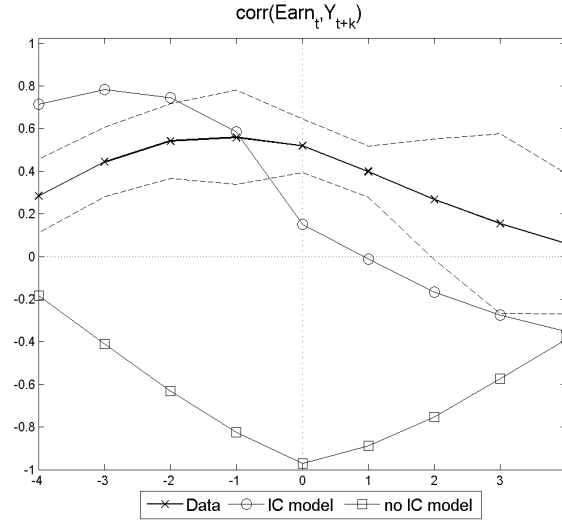


FIG. 5: Cross-correlation between Earning and Output. Note that the line with "x" markers refers to the cross-correlation coefficients from data; the circled line to those from the *IC* model; the line with "□" markers to those from *no-IC* model (II).

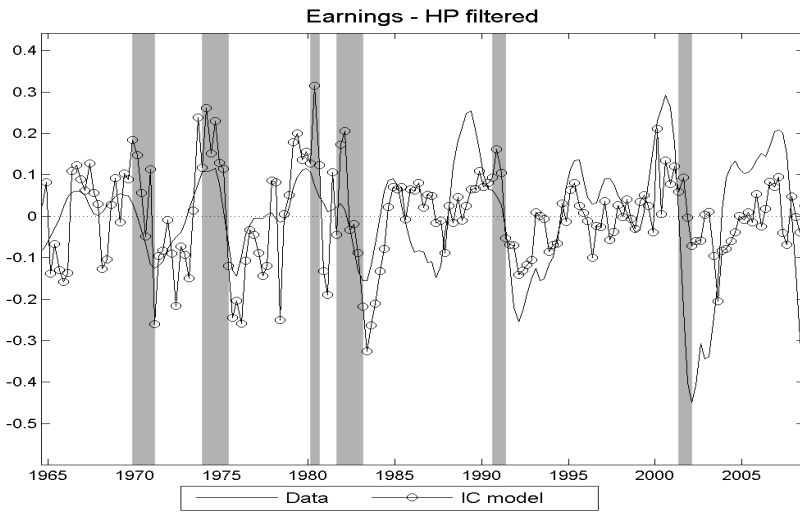


FIG. 6: Earning Dynamics: Simulation Vs Data. Note that the solid line refers to the corporate earning data and the circled line to the simulated series from *IC* model. Both series are detrended by the HP filter. The shaded vertical areas refer to the NBER recession periods.

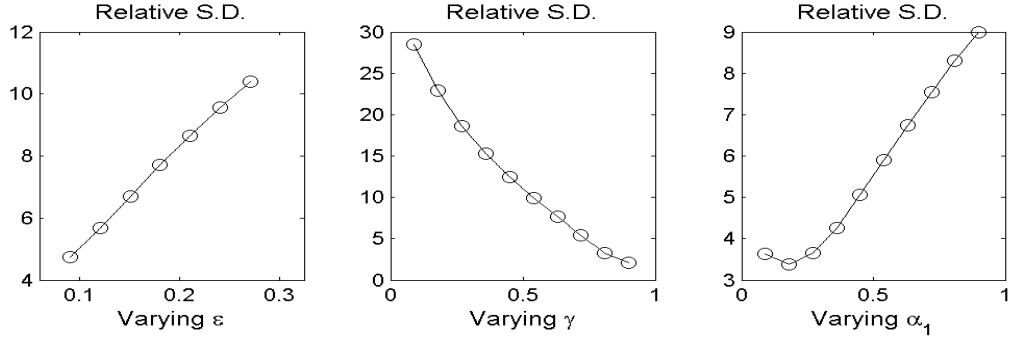


FIG. 7: Sensitivity Analysis: relative volatility of earnings with respect to output

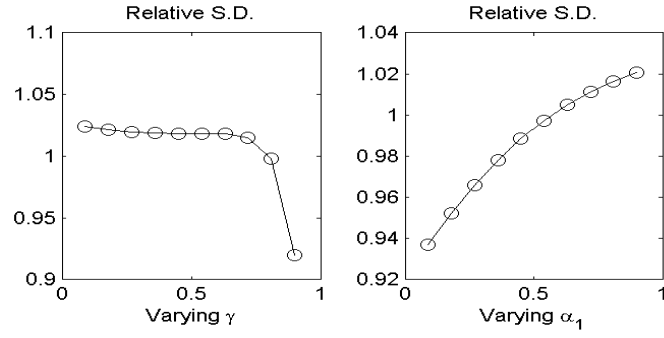


FIG. 8: Sensitivity Analysis: relative volatility of investment in IC with respect to output

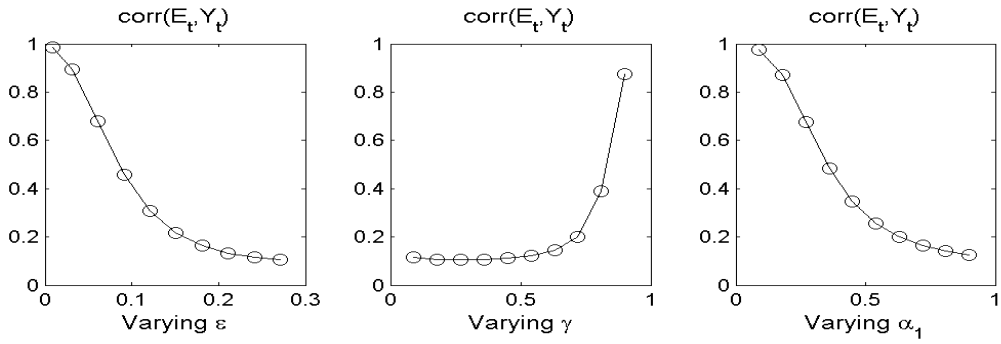


FIG. 9: Sensitivity Analysis: contemporaneous correlation of earnings with respect to output

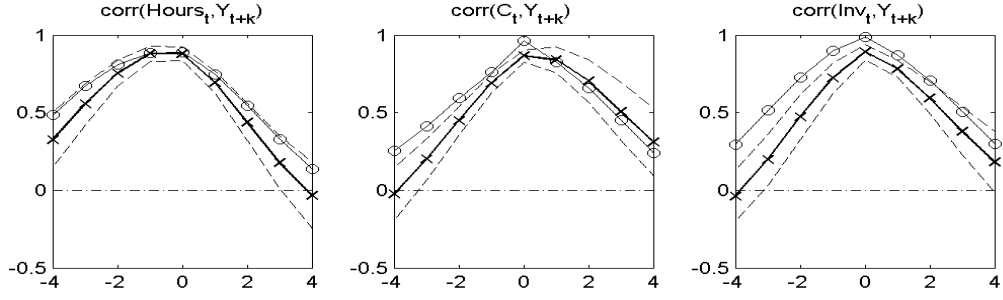


FIG. 10: Lead-Lag Cross Correlation: IC model vs Data

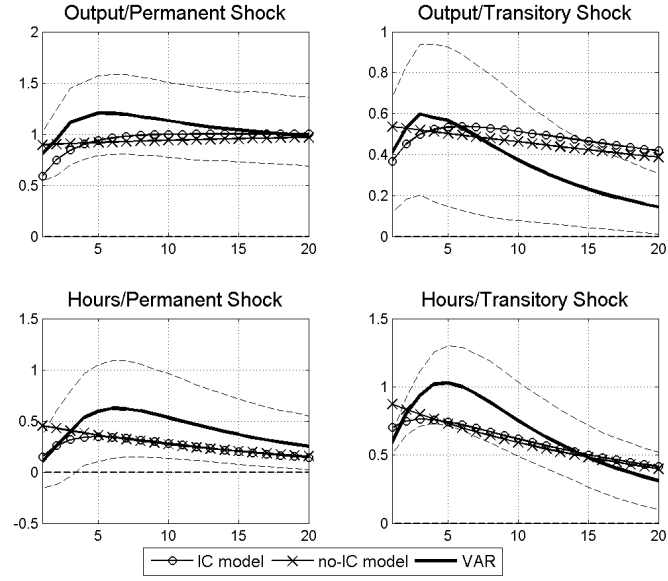


FIG. 11: Impulse Response Function (posterior mean)

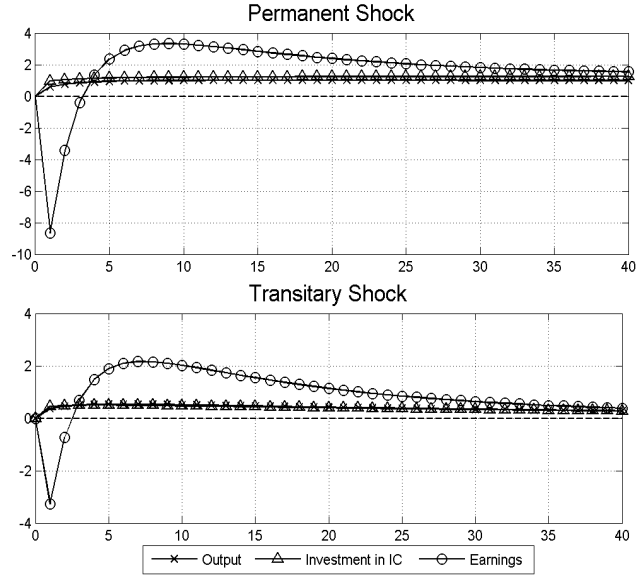


FIG. 12: Impulse responses: earnings, output and investments in intangible capital

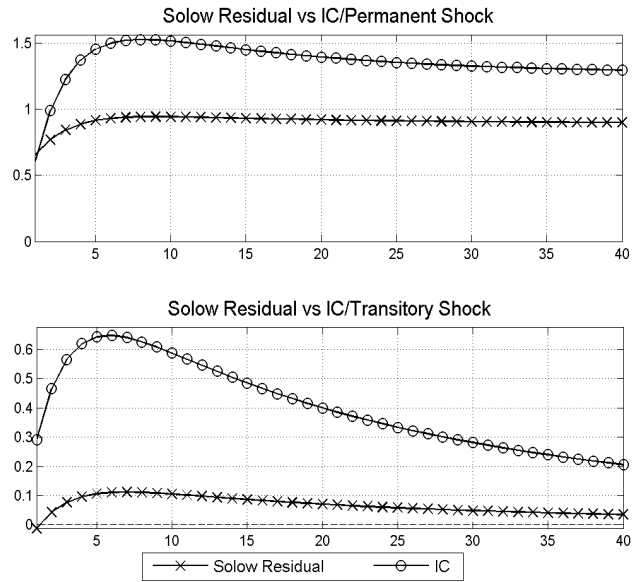


FIG. 13: Impulse Response (posterior mean): Solow Residual vs. IC

A Appendix

A.1 Sensitivity Analysis - for the Referee's Benefit

In this section, we discuss how key *IC* model parameters influence the impulse responses from the *IC* model. Figure 14 captures the role of the parameter ε as it varies while the rest of the parameters in the *IC* model remain the same as before. In all cases explored below, impulse responses from the *IC* model to both technology and preference shocks are shown.

The results indicate that as intangible capital becomes more important in production of the final good, the responses of output and hours worked display more inertia. This occurs because the desire of the firm to acquire intangible capital increases as the parameter rises. This leads to a larger diversion of resources away from production when the shocks hit the economy. As a result, productivity falls more on impact, the higher is ε . Since productivity will rise in the near future, firms shift the hiring of labour forward in time leading to a smaller increase in hours worked and output on impact.

Figure 15 shows the impulse response functions as the value of γ progressively increases towards unity. As might be expected, based on our earlier discussion, the higher is γ , the closer are the responses to the *no-IC* model. Figure 16 and 17 show the impulse response functions of earnings and investment in IC when the values of ε and γ increase towards unity, respectively. The results regarding earnings in Figure 16 and 17 reinforce the features of earning dynamics shown in Figure 7. The higher is ε , the more pronounced are the responses of earnings. Note as well that the pattern of impulse response of earning changes as ε varies. As discussed earlier, the larger is ε , the greater importance of the intertemporal trade-off for firms' earnings. A firm is more willing to give up current profits to invest in intangible capital when

ε gets bigger. On the contrary, the higher is γ , the less valuable is the current investment in intangible capital to future intangible capital stock and therefore the more dampened are the responses of earnings. As shown in the lower panels of Figure 16 and 17, there is strong correspondence between earnings and investment in IC responses. Apparently, when firms raise investment in IC, this comes at the expense of reduced current profits but results in much higher profits in the quarters that follow. We end our sensitivity analysis by noting that the sensitivity of relative volatility and comovement of earnings were discussed earlier in Figure 8.

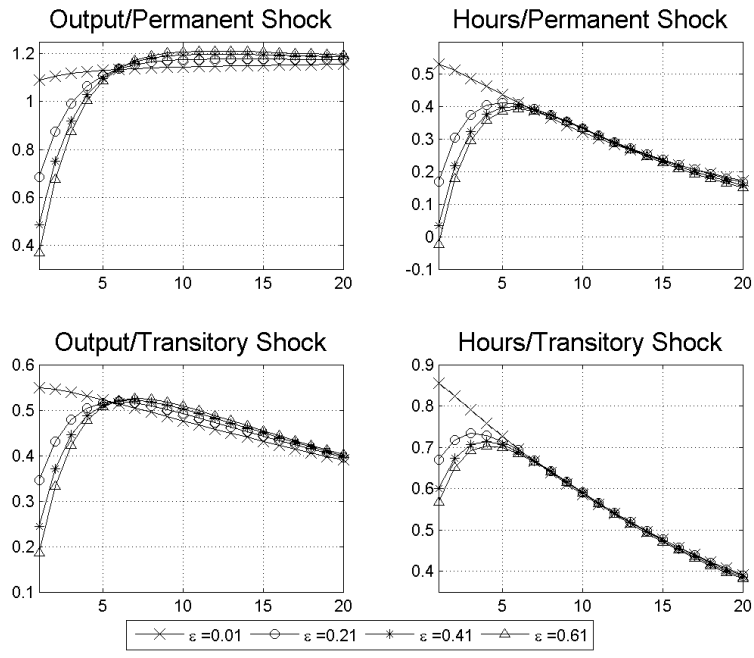


FIG. 14: Sensitivity Analysis: Impulse responses of output and hours with ϵ varying

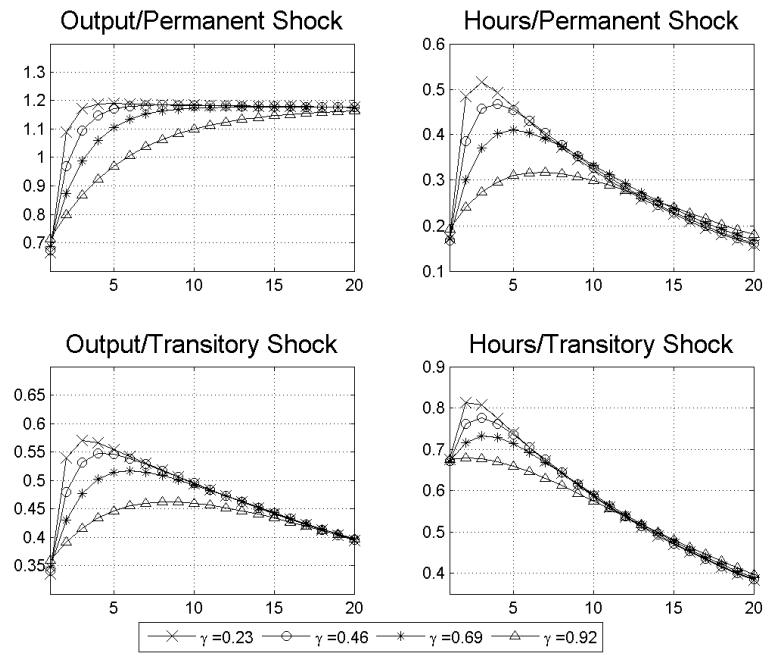


FIG. 15: Sensitivity Analysis: Impulse responses of output and hours with γ varying

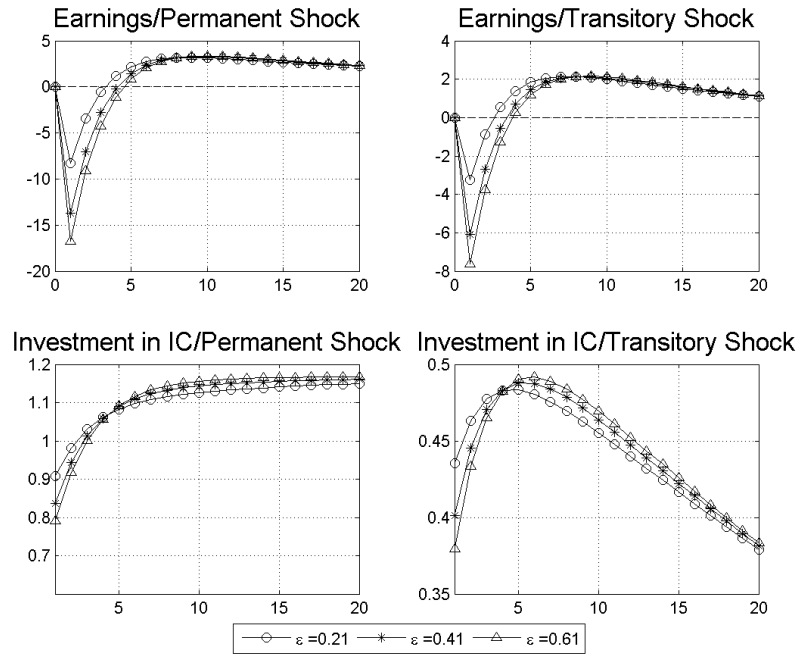


FIG. 16: Sensitivity Analysis: Impulse responses of earnings with ε varying

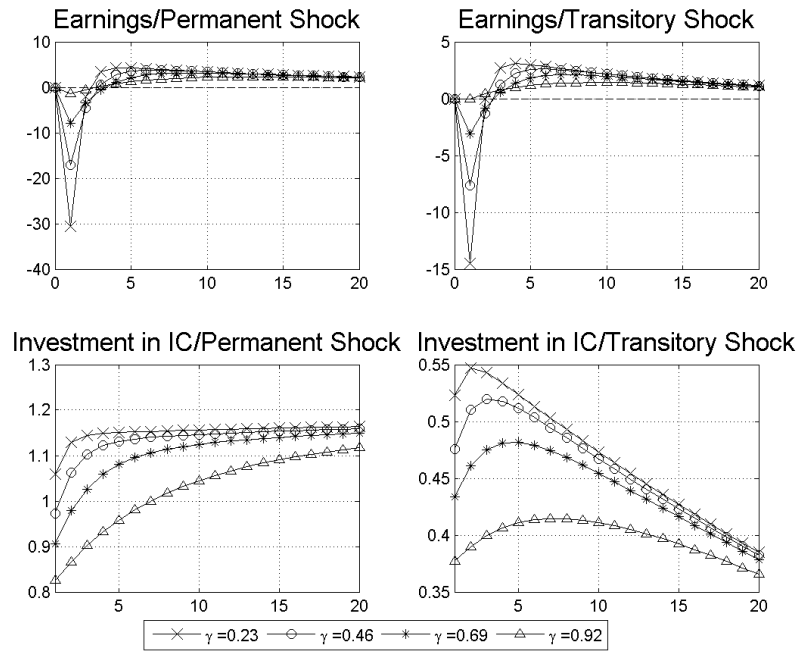


FIG. 17: Sensitivity Analysis: Impulse responses of earnings with γ varying